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A method for solving unbalanced intuitionistic fuzzy transportation problems

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Abstract: In conventional transportation problem (TP), supplies, demands and costs are always certain. This paper develops an approach to solve the unbalanced transportation problem where as all the parameters are not in deterministic numbers but imprecise ones. Here, all the parameters of the TP are considered to the triangular intuitionistic fuzzy numbers (TIFNs). The existing ranking procedure of Varghese and Kuriakose is used to transform the unbalanced intuitionistic fuzzy transportation problem (UIFTP) into a crisp one so that the conventional method may be applied to solve the TP. The occupied cells of unbalanced crisp TP that we obtained are as same as the occupied cells of UIFTP.

On the basis of this idea the solution procedure is differs from unbalanced crisp TP to UIFTP in allocation step only. Therefore, the new method and new multiplication operation on triangular intuitionistic fuzzy number (TIFN) is proposed to find the optimal solution in terms of TIFN. The main advantage of this method is computationally very simple, easy to understand and also the optimum objective value obtained by our method is physically meaningful.
Keywords: Intuitionistic fuzzy set, Triangular intuitionistic fuzzy number, Unbalanced intuitionistic fuzzy transportation problem, PSK method, Optimal solution.
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1 Introduction

In several real life situations, there is need of shipping the product from different origins (sources) to different destinations and the aim of the decision maker is to find how much quantity of the product from which origin to which destination should be shipped so that all the supply points are fully used and all the demand points are fully received as well as total transportation cost is minimum for a minimization problem or total transportation profit is maximum for a maximization problem.

Since, in real life problems, there is always existence of impreciseness in the parameters of transportation problem and in the literature Atanssov (1983), pointed out that it is better to use intuitionistic fuzzy set as compared to fuzzy set Zadeh (1965) to deal with impreciseness. The major advantage of Intuitionistic Fuzzy Set (IFS) over fuzzy set is that Intuitionistic Fuzzy Sets (IFSs) separate the degree of membership (belongingness) and the degree of non membership (non belongingness) of an element in the set. In the history of Mathematics, Burillo et al. (1994) proposed definition of intuitionistic fuzzy number and studied its properties.

An Introduction to Operations Research Taha (2008) deals the transportation problem where all the parameters are crisp number. In the lack of uncertainty of a crisp transportation problem, several researchers like Dinagar and Palanivel (2009) investigated the transportation problem in fuzzy environment using trapezoidal fuzzy numbers. Pandian and Natarajan (2010) proposed a new algorithm for finding a fuzzy optimal solution for fuzzy transportation problem where all the parameters are trapezoidal fuzzy numbers. Mohideen and Kumar (2010) did a comparative study on transportation problem in fuzzy environment. In the issues of fuzzy transportation problem, several researchers like Hussain and Kumar (2012a, 2012b), Gani and Abbas (2012) proposed a method for solving transportation problem in which all the parameters except transportation cost are represented by TIFN. Antony et al. (2014) discussed the transportation problem using triangular fuzzy number. They consider triangular fuzzy number as triangular intuitionistic fuzzy number by using format only but they are always triangular fuzzy number. Dinagar and Thiripurasundari (2014) solved transportation problem taking all the parameters that are trapezoidal intuitionistic fuzzy numbers.

Therefore, the number of researchers have been solved intuitionistic fuzzy transportation problems. To the best of our knowledge, no one has to proposed the algorithm for solving unbalanced intuitionistic fuzzy transportation problems. In this paper, we introduced a simple method called PSK method and proved a theorem, which states that every solution obtained by PSK method to fully intuitionistic fuzzy transportation problem with equality constraints is a fully intuitionistic fuzzy optimal. Atanassov (1995) presented ideas for intuitionistic fuzzy equations,inequalities and optimization. It is formulated the problem how to use the apparatus of the IFSs for the needs of optimization. Hence, this paper gives a particular answer to this problem. Moreover, the total optimum cost obtained by our method remains same as that of obtained by converting the total intuitionistic fuzzy transportation cost by applying the ranking method of Varghese and Kuriakose (2012). It is much easier to apply the proposed method as compared to the existing method.

Rest of the paper is organized as follows: Section 2 deals with some terminology and new multiplication operation, Section 3 consists of ranking procedure and ordering principles of triangular intuitionistic fuzzy number. Section 4 provides the definition of IFTP and its mathematical formulation, Section 5 deals with solution procedure, finally the conclusion is given in Section 6.

2 Preliminaries

Definition 2.1. *Let A be a classical set, $\mu_A(x)$ be a function from A to $[0,1]$. A fuzzy set A^* with the membership function $\mu_A(x)$ is defined by*

$$A^* = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1]\}.$$

Definition 2.2. *Let X be denote a universe of discourse, then an intuitionistic fuzzy set A in X is given by a set of ordered triples,*

$$\tilde{A}^I = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

Where $\mu_A, \nu_A : X \to [0,1]$, are functions such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$. For each x the membership $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and the degree of non-membership of the element $x \in X$ to $A \subset X$ respectively.

Definition 2.3. *An Intuitionistic fuzzy subset $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ of the real line R is called an intuitionistic fuzzy number (IFN) if the following holds:*

i. There exist $m \in R, \mu_A(m) = 1$ and $\nu_A(m) = 0$, (m is called the mean value of A).

ii. μ_A is a continuous mapping from R to the closed interval $[0,1]$ and $\forall x \in R$, the relation $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ holds.

The membership and non membership function of A is of the following form:

$$\mu_A(x) = \begin{cases} 0 & for\ -\infty < x \leq m - \alpha \\ f_1(x) & for\ x \in [m-\alpha, m] \\ 1 & for\ x = m \\ h_1(x) & for\ x \in [m, m+\beta] \\ 0 & for\ m+\beta \leq x < \infty \end{cases}$$

Where $f_1(x)$ and $h_1(x)$ are strictly increasing and decreasing function in $[m-\alpha, m]$ and $[m, m+\beta]$ respectively.

$$\nu_A(x) = \begin{cases} 1 & for\ -\infty < x \leq m - \alpha' \\ f_2(x) & for\ x \in [m-\alpha', m]; 0 \leq f_1(x) + f_2(x) \leq 1 \\ 0 & for\ x = m \\ h_2(x) & for\ x \in [m, m+\beta']; 0 \leq h_1(x) + h_2(x) \leq 1 \\ 1 & for\ m+\beta' \leq x < \infty \end{cases}$$

Here m is the mean value of $A.\alpha, \beta$ are called left and right spreads of membership function $\mu_A(x)$ respectively. α', β' are represents left and right spreads of non membership function $\nu_A(x)$ respectively. Symbolically, the intuitionistic fuzzy number $\tilde{A}^I$ is represented as $A_{IFN} = (m; \alpha, \beta; \alpha', \beta')$.

Definition 2.4. *A fuzzy number A is defined to be a triangular fuzzy number if its membership function $\mu_A : \mathbb{R} \to [0,1]$ is equal to*

$$\mu_A(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)} & \text{if } x \in [a_1, a_2] \\ \frac{(a_3-x)}{(a_3-a_2)} & \text{if } x \in [a_2, a_3] \\ 0 & otherwise \end{cases}$$

where $a_1 \leq a_2 \leq a_3$. This fuzzy number is denoted by (a_1, a_2, a_3).

Definition 2.5. *A Triangular Intuitionistic Fuzzy Number($\tilde{A}^I$ is an intuitionistic fuzzy set in R with the following membership function $\mu_A(x)$ and non-membership function $\nu_A(x)$:)*

$$\mu_A(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \frac{(x-a_1)}{(a_2-a_1)} & \text{for } a_1 \leq x \leq a_2 \\ 1 & \text{for } x = a_2 \\ \frac{(a_3-x)}{(a_3-a_2)} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{for } x > a_3 \end{cases} \qquad \nu_A(x) = \begin{cases} 1 & \text{for } x < a_1' \\ \frac{(a_2-x)}{(a_2-a_1')} & \text{for } a_1' \leq x \leq a_2 \\ 0 & \text{for } x = a_2 \\ \frac{(x-a_2)}{(a_3'-a_2)} & \text{for } a_2 \leq x \leq a_3' \\ 1 & \text{for } x > a_3' \end{cases}$$

Where $a_1^{'} \leq a_1 \leq a_2 \leq a_3 \leq a_3^{'}$ and $\mu_A(x), \nu_A(x) \leq 0.5$ for $\mu_A(x) = \nu_A(x), \forall x \in \mathbb{R}$ This TIFN is denoted by $\tilde{A}^I = (a_1, a_2, a_3)(a_1^{'}, a_2, a_3^{'})$

Particular cases: Let $\tilde{A}^I = (a_1, a_2, a_3)(a_1^{'}, a_2, a_3^{'})$ be a TIFN. Then the following cases arise
Case 1: If $a_1^{'} = a_1, a_3^{'} = a_3$, then $\tilde{A}^I$ represent Tringular Fuzzy Number (TFN). It is denoted by $A = (a_1, a_2, a_3)$.
Case 2: If $a_1^{'} = a_1 = a_2 = a_3 = a_3^{'} = m$,then $\tilde{A}^I$ represent a real number m.

Definition 2.6. *Let $\tilde{A}^I = (a_1, a_2, a_3)(a_1^{'}, a_2, a_3^{'})$ and $\tilde{B}^I = (b_1, b_2, b_3)(b_1^{'}, b_2, b_3^{'})$ be any two TIFNs then the arithmetic operations as follows.*
Addition: $\tilde{A}^I \oplus \tilde{B}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3)(a_1^{'} + b_1^{'}, a_2 + b_2, a_3^{'} + b_3^{'})$
Subtraction: $\tilde{A}^I \ominus \tilde{B}^I = (a_1 - b_3, a_2 - b_2, a_3 - b_1)(a_1^{'} - b_3^{'}, a_2 - b_2, a_3^{'} - b_1^{'})$
Multiplication: *If $\Re(\tilde{A}^I), \Re(\tilde{B}^I) \geq 0$, then*
$\tilde{A}^I \otimes \tilde{B}^I = (a_1\Re(\tilde{B}^I), a_2\Re(\tilde{B}^I), a_3\Re(\tilde{B}^I))(a_1'\Re(\tilde{B}^I), a_2\Re(\tilde{B}^I), a_3'\Re(\tilde{B}^I))$

Note 1. *All the parameters of the conventional TP such as supplies, demands and costs are in positive. Since in transportation problems negative parameters has no physical meaning. Hence, in the proposed method all the parameters may be assumed as non-negative triangular intuitionistic fuzzy number. On the basis of this idea we need not further investigate the multiplication operation under the condition that $\Re(\tilde{A}^I), \Re(\tilde{B}^I) < 0$.*

Scalar Multiplication:

i. $k\tilde{A}^I = (ka_1, ka_2, ka_3)(ka_1^{'}, ka_2, ka_3^{'}),$ for $k \geq 0$

ii. $k\tilde{A}^I = (ka_3, ka_2, ka_1)(ka_3^{'}, ka_2, ka_1^{'}),$ for $k < 0$

3 Comparison of TIFN

Let $\tilde{A}^I = (a_1, a_2, a_3)(a_1^{'}, a_2, a_3^{'})$ and $\tilde{B}^I = (b_1, b_2, b_3)(b_1^{'}, b_2, b_3^{'})$ be two TIFNs. Then
(i) $\Re(\tilde{A}^I) > \Re(\tilde{B}^I)$ iff $\tilde{A}^I \succ \tilde{B}^I$
(ii) $\Re(\tilde{A}^I) < \Re(\tilde{B}^I)$ iff $\tilde{A}^I \prec \tilde{B}^I$
(iii) $\Re(\tilde{A}^I) = \Re(\tilde{B}^I)$ iff $\tilde{A}^I \approx \tilde{B}^I$. where,

$$\Re(\tilde{A}^I) = \frac{1}{3}\left[\frac{(a_3^{'} - a_1^{'})(a_2 - 2a_3^{'} - 2a_1^{'}) + (a_3 - a_1)(a_1 + a_2 + a_3) + 3(a_3^{'2} - a_1^{'2})}{a_3^{'} - a_1{'} + a_3 - a_1}\right]$$

$$\Re(\tilde{B}^I) = \frac{1}{3}\left[\frac{(b_3^{'} - b_1^{'})(b_2 - 2b_3^{'} - 2b_1^{'}) + (b_3 - b_1)(b_1 + b_2 + b_3) + 3(b_3^{'2} - b_1^{'2})}{b_3^{'} - b_1{'} + b_3 - b_1}\right]$$

4 Fully intuitionistic fuzzy transportation problem and its mathematical formulation

Definition 4.1. *If the transportation problem has at least one of the parameter (cost) or two of the parameters (supply and demand) or all of the parameters (supply, demand and cost) in intuitionistic fuzzy numbers then the problem is called IFTP. Otherwise it is not an IFTP.*
Further, intuitionistic fuzzy transportation problem can be classified into four categories. They are:

- *Type-1 intuitionistic fuzzy transportation problems;*
- *Type-2 intuitionistic fuzzy transportation problem;*
- *Type-3 intuitionistic fuzzy transportation problem (Mixed Intuitionistic Fuzzy Transportation Problem);*
- *Type-4 intuitionistic fuzzy transportation problem (Fully Intuitionistic Fuzzy Transportation Problem).*

Definition 4.2. *A transportation problem having intuitionistic fuzzy availabilities and intuitionistic fuzzy demands but crisp costs is termed as intuitionistic fuzzy transportation problem of type-1.*

Definition 4.3. *A transportation problem having crisp availabilities and crisp demands but intuitionistic fuzzy costs is termed as intuitionistic fuzzy transportation problem of type-2.*

Definition 4.4. *The transportation problem is said to be the type-3 intuitionistic fuzzy transportation problem or mixed intuitionistic fuzzy transportation problem if all the parameters of the transportation problem (such as supplies, demands and costs) must be in the mixture of crisp numbers, triangular fuzzy numbers and triangular intuitionistic fuzzy numbers.*

Definition 4.5. *The transportation problem is said to be the type-4 intuitionistic fuzzy transportation problem or fully intuitionistic fuzzy transportation problem if all the parameters of the transportation problem (such as supplies, demands and costs) must be in intuitionistic fuzzy numbers.*

Definition 4.6. *The transportation problem is said to be balanced intuitionistic fuzzy transportation problem if total intuitionistic fuzzy supply is equal to total intuitionistic fuzzy demand. That is,* $\sum_{i=1}^{m} \tilde{a}_i^I = \sum_{j=1}^{n} \tilde{b}_j^I$

Definition 4.7. *The transportation problem is said to be an unbalanced intuitionistic fuzzy transportation problem if total intuitionistic fuzzy supply is not equal to total intuitionistic fuzzy demand. That is,* $\sum_{i=1}^{m} \tilde{a}_i^I \neq \sum_{j=1}^{n} \tilde{b}_j^I$

Definition 4.8. *A set of intuitionistic fuzzy non negative allocations* $\tilde{x}_{ij}^I > \tilde{0}^I$ *satisfies the row and column restriction is known as intuitionistic fuzzy feasible solution.*

Definition 4.9. *Any feasible solution is an intuitionistic fuzzy basic feasible solution if the number of non negative allocations is at most* $(m + n - 1)$ *where* m *is the number of rows and* n *is the number of columns in the* $m \times n$ *transportation table.*

Definition 4.10. *If the intuitionistic fuzzy basic feasible solution contains less than* $(m + n - 1)$ *non negative allocations in* $m \times n$ *transportation table, it is said to be degenerate.*

Definition 4.11. *Any intuitionist fuzzy feasible solution to a transportation problem containing m origins and n destinations is said to be intuitionist fuzzy non degenerate, if it contains exactly* $(m + n - 1)$ *occupied cells.*

Definition 4.12. *The intuitionistic fuzzy basic feasible solution is said to be intuitionistic fuzzy optimal solution if it minimizes the total intuitionistic fuzzy transportation cost (or) it maximizes the total intuitionistic fuzzy transportation profit.*

Mathematical Formulation: Consider the transportation problem with m origins (rows) and n destinations (columns). Let $\tilde{c}_{ij}^I$ be the cost of transporting one unit of the product from i^{th} origin to j^{th} destination. Let $\tilde{a}_i^I$ be the quantity of commodity available at origin i. Let $\tilde{b}_j^I$ be the quantity of commodity needed at destination j and $\tilde{x}_{ij}^I$ is the quantity transported from i^{th} origin to j^{th} destination, so as to minimize the total intuitionistic fuzzy transportation cost.

Mathematically Fully Intuitionistic Fuzzy Transportation Problem can be stated as (FIFTP).

(Model 1) (P)Minimize $\tilde{Z}^I = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}^I \otimes \tilde{x}_{ij}^I$

$$\text{Subject to,} \quad \begin{aligned} \sum_{j=1}^{n} \tilde{x}_{ij}^I &\approx \tilde{a}_i^I, \quad \text{for } i = 1, 2, \cdots, m \\ \sum_{i=1}^{m} \tilde{x}_{ij}^I &\approx \tilde{b}_j^I, \quad \text{for } j = 1, 2, \cdots, n \\ \tilde{x}_{ij}^I &\succcurlyeq \tilde{0}^I, \quad \text{for } i = 1, 2, \cdots, m \text{ and } j = 1, 2, \cdots, n \end{aligned}$$

where $m =$ the number of supply points, $n =$ the number of demand points:

$$\tilde{c}^I_{ij} = (c^1_{ij}, c^2_{ij}, c^3_{ij})(c^{1'}_{ij}, c^2_{ij}, c^{3'}_{ij}), \tilde{a}^I_i = (a^1_i, a^2_i, a^3_i)(a^{1'}_i, a^2_i, a^{3'}_i),$$

$$\tilde{b}^I_j = (b^1_j, b^2_j, b^3_j)(b^{1'}_j, b^2_j, b^{3'}_j), \tilde{x}^I_{ij} = (x^1_{ij}, x^2_{ij}, x^3_{ij})(x^{1'}_{ij}, x^2_{ij}, x^{3'}_{ij}).$$

When the supplies, demands and costs are intuitionistic fuzzy numbers, then the total cost becomes an intuitionistic fuzzy number.

$$\tilde{Z}^I = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}^I_{ij} \otimes \tilde{x}^I_{ij}$$

Hence it cannot be minimized directly. For solving the problem we convert the intuitionistic fuzzy supplies, intuitionistic fuzzy demands and the intuitionistic fuzzy costs into crisp ones by an intuitionistc fuzzy number ranking method of Varghese and Kuriakose.

Consider the transportation problem with m origins (rows) and n destinations (columns). Let c_{ij} be the cost of transporting one unit of the product from i^{th} origin to j^{th} destination. Let a_i be the quantity of commodity available at origin i. Let b_j the quantity of commodity needed at destination j. Let x_{ij} denote the quantity of commodity transported from i^{th} origin to j^{th} destination, so as to minimize the total transportation cost.

(Model 2) (TP)Minimize $\Re(\tilde{Z}^{I*}) = \sum_{i=1}^{m}\sum_{j=1}^{n} \Re(\tilde{c}^I_{ij}) \otimes \Re(\tilde{x}^I_{ij})$

$$\begin{array}{rrcll} \text{Subject to,} & \sum_{j=1}^{n} \Re(\tilde{x}^I_{ij}) & \approx & \Re(\tilde{a}^I_i), & \text{for } i = 1, 2, \cdots, m \\ & \sum_{i=1}^{m} \Re(\tilde{x}^I_{ij}) & \approx & \Re(\tilde{b}^I_j), & \text{for } j = 1, 2, \cdots, n \\ & \Re(\tilde{x}^I_{ij}) & \succcurlyeq & \Re(\tilde{0}^I), & \text{for } i = 1, 2, \cdots, m \text{ and } j = 1, 2, \cdots, n \end{array}$$

Since $\Re(\tilde{c}^I_{ij}), \Re(\tilde{a}^I_i)$ and $\Re(\tilde{b}^I_j)$ all are crisp values, this problem (2) is obviously the crisp transportation problem of the form (1) which can be solved by the conventional method namely the Zero Point Method, Modified Distribution Method or any other software package such as TORA , LINGO and so on. Once the optimal solution x^* of Model (2) is found, the optimal intuitionistic fuzzy objective value $\tilde{Z}^{I*}$ of the original problem can be calculated as

$$\tilde{Z}^{I*} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}^I_{ij} \otimes \tilde{x}^{I*}_{ij}$$

where, $\tilde{c}^I_{ij} = (c^1_{ij}, c^2_{ij}, c^3_{ij})(c^{1'}_{ij}, c^2_{ij}, c^{3'}_{ij}), \tilde{x}^{I*}_{ij} = (x^1_{ij}, x^2_{ij}, x^3_{ij})(x^{1'}_{ij}, x^2_{ij}, x^{3'}_{ij})$.

The above FIFTP and its equivalent crisp TP can be stated in the below tabular form as follows:

Table 1: Tabular representation of FIFTP

Source	Destination				Intuitionistic fuzzy supply
	D_1	D_2	$\cdots$	D_n	$\tilde{a}_i^I$
S_1	$\tilde{c}_{11}^I$	$\tilde{c}_{12}^I$	$\ldots$	$\tilde{c}_{1n}^I$	$\tilde{a}_1^I$
S_2	$\tilde{c}_{21}^I$	$\tilde{c}_{22}^I$	$\ldots$	$\tilde{c}_{2n}^I$	$\tilde{a}_2^I$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	$\tilde{c}_{m1}^I$	$\tilde{c}_{m2}^I$	$\ldots$	$\tilde{c}_{mn}^I$	$\tilde{a}_m^I$
Intuitionistic fuzzy demand $\tilde{b}_j^I$	$\tilde{b}_1^I$	$\tilde{b}_2^I$	$\ldots$	$\tilde{b}_n^I$	$\sum_{i=1}^{m} \tilde{a}_i^I = \sum_{j=1}^{n} \tilde{b}_j^I$

Table 2: Tabular representation of crisp TP

Source	Destination				Intuitionistic fuzzy supply
	D_1	D_2	$\cdots$	D_n	$\tilde{a}_i^I$
S_1	$\Re(\tilde{c}_{11}^I)$	$\Re(\tilde{c}_{12}^I)$	$\ldots$	$\Re(\tilde{c}_{1n}^I)$	$\Re(\tilde{a}_1^I)$
S_2	$\Re(\tilde{c}_{21}^I)$	$\Re(\tilde{c}_{22}^I)$	$\ldots$	$\Re(\tilde{c}_{2n}^I)$	$\Re(\tilde{a}_2^I)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	$\Re(\tilde{c}_{m1}^I)$	$\Re(\tilde{c}_{m2}^I)$	$\ldots$	$\Re(\tilde{c}_{mn}^I)$	$\Re(\tilde{a}_m^I)$
Intuitionistic fuzzy demand $\Re(\tilde{b}_j^I)$	$\Re(\tilde{b}_1^I)$	$\Re(\tilde{b}_2^I)$	$\ldots$	$\Re(\tilde{b}_n^I))$	$\sum_{i=1}^{m} \Re(\tilde{a}_i^I) = \sum_{j=1}^{n} \Re(\tilde{b}_j^I)$

5 PSK method

Step 1. Consider the TP having all the parameters such as supplies, demands and costs must be in intuitionistic fuzzy numbers (This situation is known as FIFTP).

Step 2. Examine whether the total intuitionistic fuzzy supply equals total intuitionistic fuzzy demand. If not, introduce a dummy row/column having all its cost elements as intuitionistic fuzzy zero and intuitionistic fuzzy supply/intuitionistic fuzzy demand as the positive difference of intuitionistic fuzzy supply and intuitionistic fuzzy demand.

Step 3. After using step 2, transform the FIFTP into its equivalent crisp TP using the ranking procedure as mentioned in section 3.

Step 4. Now, the crisp TP having all the entries of supplies, demands and costs are integers then kept as it is. Otherwise at least one or all of the supplies, demands and costs are not in integers then rewrite its nearest integer value.

Step 5. After using step 4 of the proposed method, now solve the crisp TP by using any one of the existing methods (MODI, Zero Point Method) or software packages such as TORA, LINGO and so on. This step yields the optimal allocation and optimal objective value of the crisp TP (This optimal allotted cells in crisp transportation table is referred as occupied cells).

Step 6. After using step 5 of the proposed method, now check if one or more rows/columns having exactly one occupied cell then allot the maximum possible value in that cell and adjust the corresponding supply or demand with a positive difference of supply and demand. Otherwise, if all the rows/columns having more than one occupied cells then select a cell in the α- row and β- column of the transportation table whose cost is maximum (If the maximum cost is more than one i.e, a tie occurs then select arbitrarily) and examine which one of the cell is minimum cost (If the minimum cost is more than one i.e, a tie occurs then select arbitrarily) among all the occupied cells in that row and column then allot the maximum possible value to that cell. In this manner proceeds selected row and column entirely. If the entire row and column of the occupied cells having fully allotted then select the next maximum cost of the transportation table and examine which one of the cell is minimum cost among all the occupied cells in that row and column then allot the maximum possible value to that cell. Repeat this process until all the intuitionistic fuzzy supply points are fully used and all the intuitionistic fuzzy demand points are fully received. This step yields the fully intuitionistic fuzzy solution to the given FIFTP.

Note 2. *Allot the maximum possible value to the occupied cells in FIFTP which is the most preferable row/column having exactly one occupied cell.*

Theorem 5.1. *A solution obtained by the PSK method for a fully intuitionistic fuzzy transportation problem with equality constraints (P) is a fully intuitionistic fuzzy optimal solution for the fully intuitionistic fuzzy transportation problem (P).*

Proof. We, now describe the PSK method in detail. We construct the fully intuitionistic fuzzy transportation table$[\tilde{c}^I_{ij}]$ for the given fully intuitionistic fuzzy transportation problem and then, convert it into a balanced one if it is not balanced. Now, transform the FIFTP into its equivalent crisp TP using the ranking procedure of Varghese and Kuriakose.

Now, the crisp TP having all the entries of supplies, demands and costs are integers then kept as it is. Otherwise at least one or all of the supplies, demands and costs are not in integers then rewrite its nearest integer value because decimal values in transportation problem has no physical meaning (such a transportation problem referred as crisp TP).

Now, solve the crisp TP by using any one of the existing methods (MODI, Zero Point Method) or software packages such as TORA, LINGO and so on. This step yields the optimal allotment and optimal objective value of the crisp TP (The optimal allotted cells in crisp transportation table is referred to as occupied cells which are at most $m + n - 1$ and all are basic feasible. The remaining cells are called unoccupied cells which are all none basic at zero level).

Clearly, occupied cells in crisp TP is same as that of occupied cells in FIFTP. Therefore, we need not further investigate the occupied cells in FIFTP. But only we claim that how much quantity to allot the occupied cells under without affecting the rim requirements.

Now, check if one or more rows/columns having exactly one occupied cell then allot the maximum possible value in that cell and adjust the corresponding supply or demand with a positive difference of supply and demand. Otherwise, if all the rows/columns having more than one occupied cells then select a cell in the α- row and β- column of the transportation table whose cost is maximum (If the maximum cost is more than one i.e, a tie occurs then select arbitrarily) and examine which one of the cell is minimum cost (If the minimum cost is more than one i.e, a tie occurs then select arbitrarily) among all the occupied cells in that row and column then allot the maximum possible value to that cell. In this manner proceeds selected row/column entirely. If the entire row and column of the occupied cells having fully allotted then select the next maximum cost of the transportation table and examine which one of the cell is minimum cost among all the occupied cells in that row and column then allot the maximum possible value to that cell. Repeat this process until all the intuitionistic fuzzy supply points are fully used and all the intuitionistic fuzzy demand points are fully received. This step yields the optimum intuitionistic fuzzy allotment.

Clearly, the above process will satisfy all the rim requirements (row and column sum restriction). If all the rim requirements is satisfied then automatically it satisfied total intuitionistic fuzzy supply is equal to total intuitionistic fuzzy demand i.e., the necessary and sufficient condition for the FIFTP is satisfied.

Hence, the solution obtained by the PSK method for a fully intuitionistic fuzzy transportation problem with equality constraints is a fully intuitionistic fuzzy optimal solution for the fully intuitionistic fuzzy transportation problem. □

Advantages of the proposed method

By using the proposed method a decision maker has the following advantages:

i. The optimum objective value of the unbalanced IFTP is non-negative triangular intuitionistic fuzzy number i.e., there is no negative part in the obtained triangular intuitionistic fuzzy number.

ii. The proposed method is computationally very simple and easy to understand.

6 Conclusion

On the basis of the present study, it can be concluded that the UIFTP, IFTP and FIFTP which can be solved by the existing methods (Hussain and Kumar (2012), Gani and Abbas (2013), Antony et al. (2014), Dinagar and Thiripurasundari (2014)) can also be solved by the proposed method. The UIFTP with equality constraints is solved by the PSK method which differs from the existing methods namely, intuitionistic fuzzy modified distribution method, intuitionistic fuzzy zero point method, zero suffix method, revised distribution method and average method. The main advantage of this method is computationally very simple, easy to understand and the solution obtained by this method is always optimal.The proposed method enable a decision-maker to determine resource allocation in an economic way. Also, intuitionistic fuzzy ranking method offers an effective tool for handling various types of problems like, project scheduling, network flow problems and transportation problems.

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